6.1C Properties of Exponents (Division)

6.2 Radicals and Rational Exponents

Expand. What do you think the rule is?

$$\frac{x^9}{x^3} = \frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}}$$

$$= x\cdot x\cdot x\cdot x\cdot x\cdot x$$

$$= x^6$$

Quotient Property

When dividing powers with the same bases, treat the division bar like a giant subtraction sign

* $\dfrac{x^9}{x^3} = x^{9-3} = x^6$

* $\dfrac{x^4}{x^7} = x^{4-7} = x^{-3} = \dfrac{1}{x^3}$

* Take the bigger exponent and subtract the smaller - where the big one was is where the base will stay!

Simplify.

1. $\dfrac{\cancel{2}x^2}{\cancel{2}x^1} = x^{2-1}$
   $= x^1$
   $\boxed{= x}$

2. $\dfrac{\cancel{2}n^2}{\cancel{2}n^4} = n^{2-4}$
   $= n^{-2}$
   $\boxed{= \dfrac{1}{n^2}}$

3. $\dfrac{3a^2}{a^{-1}} = 3a^{2-(-1)}$
   $= 3a^{2+1}$
   $\boxed{= 3a^3}$

4. $\dfrac{3k}{4k^2} = \dfrac{3k^{1-2}}{4}$
   $= \dfrac{3k^{-1}}{4}$
   $\boxed{= \dfrac{3}{4k}}$

5. $\dfrac{x^4}{3x^2} = \dfrac{x^{4-2}}{3}$
   $\boxed{= \dfrac{x^2}{3}}$

6. $\dfrac{3x^{-4}}{4x^4} = \dfrac{3x^{-4-4}}{4}$
   $= \dfrac{3x^{-8}}{4}$
   $\boxed{= \dfrac{3}{4x^8}}$

Simplify each.

1. $\dfrac{2x^3y^{-4}}{x^3y^2} = 2x^{3-3}y^{-4-2}$

$= 2x^0y^{-6}$

$\boxed{= \dfrac{2}{y^6}}$

2. $\dfrac{2y^{-2}}{3x^0y^3}$ (1) $= \dfrac{2}{3}y^{-2-3}$

$= \dfrac{2}{3}y^{-5}$

$\boxed{= \dfrac{2}{3y^5}}$

3. $\dfrac{{}^{1}\cancel{2}m^{-4}n^2}{{}_{2}\cancel{4}nm^4}$

$= \dfrac{m^{-4-4}n^{2-1}}{2}$

$= \dfrac{m^{-8}n^1}{2}$

$\boxed{= \dfrac{n}{2m^8}}$

4. $\dfrac{4xy^{-3}}{x^{-3}y^3}$

$=$

$= \dfrac{4x\cdot x^3}{y^3\cdot y^3}$

$\boxed{= \dfrac{4x^4}{y^6}}$

Simplify.

$\left(\dfrac{ab^4}{c^2d^3}\right)^5 = \dfrac{a^5b^{4\cdot5}}{c^{2\cdot5}d^{3\cdot5}}$

$\boxed{= \dfrac{a^5b^{20}}{c^{10}d^{15}}}$

$\left(\dfrac{a^3b}{a^2b^2}\right)^3 = \left(a^{3-2}b^{1-2}\right)^3$

$= (ab^{-1})^3$

$= \left(\dfrac{a}{b}\right)^3 = \boxed{\dfrac{a^3}{b^3}}$

$\left(\dfrac{2}{3}\right)^{-2}\left(\dfrac{\overset{3}{\cancel{6}}m}{\cancel{2}n}\right)^{-3}$

$= \left(\dfrac{2^{-2}}{3^{-2}}\right)\left(\dfrac{3^{-3}m^{-3}}{n^{-3}}\right)$

$= \left(\dfrac{3^2}{2^2}\right)\left(\dfrac{n^3}{3^3m^3}\right)$

$= \dfrac{3^2n^2}{2^23^3m^3} = \dfrac{n^2}{4\cdot3m^3} = \boxed{\dfrac{n^2}{12m^3}}$

$\left(\dfrac{2x^2}{y^3}\right)^{-2} = \left(\dfrac{2^{-2}x^{-4}}{y^{-6}}\right)$

$= \left(\dfrac{y^6}{2^2x^4}\right)$

$\boxed{= \dfrac{y^6}{4x^4}}$

Simplify each.

1. $-\dfrac{18x^3y^{-9}}{3x^2y^2} = -\dfrac{18}{3} \cdot \dfrac{x^3}{x^2} \cdot \dfrac{y^{-9}}{y^2}$

$= -6 \cdot x^{3-2}\, y^{-9-2}$

$= -6\, x^1\, y^{-11}$

$\boxed{= \dfrac{-6x}{y^{11}}}$

2. $\dfrac{5x^2y^9}{15x^4y^9} = \dfrac{5}{15} \cdot \dfrac{x^2}{x^4} \cdot \dfrac{y^9}{y^9}$

$= \dfrac{1}{3} \cdot x^{2-4} \cdot y^{9-9}$

$= \dfrac{1}{3}\, x^{-2}\, y^0$

$\boxed{= \dfrac{1}{3x^2}}$

3. $\dfrac{6a^3b^{-2}}{8ab^{-5}} = \dfrac{6}{8} \cdot \dfrac{a^3}{a} \cdot \dfrac{b^{-2}}{b^{-5}}$

$= \dfrac{3}{4} \cdot a^{3-1}\, b^{-2\cancel{+}-5}$

$= \dfrac{3}{4}\, a^2\, b^3$

$\boxed{= \dfrac{3a^2b^2}{4}}$

4. $\dfrac{12a^{-3}b^3}{16a^5b^4} = \dfrac{12}{16} \cdot \dfrac{a^{-3}}{a^5} \cdot \dfrac{b^3}{b^4}$

$= \dfrac{3}{4}\, a^{-3-5}\, b^{3-\cancel{4}}$

$= \dfrac{3}{4}\, a^{-8}\, b^{-1}$

$\boxed{= \dfrac{3}{4a^8b}}$

Simplifying Exponential Expressions

* No negative or zero exponents

* Same base does not appear more than once

* No powers raised to powers (no parentheses)

* No products raised to powers

* No quotients raised to powers

* Numerical coefficients do not have any factor in common

6.2 Radical and Rational Exponents

A number that is multiplied by itself to form a product is called a **square root** of that product. The operations of squaring and finding a square root are inverse operations.

The radical symbol $\sqrt{\ }$, is used to represent square roots. Positive real numbers have two square roots.

Example 1: Finding Square Roots of Perfect Squares

Find each square root.

A. $\sqrt{16} = \pm 4$

B. $-\sqrt{9}$

$= \pm 3$

C. $\sqrt{\frac{25}{81}} = \frac{\sqrt{25}}{\sqrt{81}} = \pm \frac{5}{9}$

An nth root of a is written $\sqrt[n]{a}$, where the expression $\sqrt[n]{a}$ is called a **radical** and n is the **index** of the radical.

index radicand

$$\sqrt[n]{a}$$

Real nth Roots of a

Let n be an integer greater than 1, and let a be a real number.

- If n is odd, then a has one real nth root: $\sqrt[n]{a} = a^{1/n}$
- If n is even and $a > 0$, then a has two real nth roots: $\pm\sqrt[n]{a} = \pm a^{1/n}$
- If n is even and $a = 0$, then a has one real nth root: $\sqrt[n]{0} = 0$
- If n is even and $a < 0$, then a has no real nth roots.

Algebra $a^{m/n} = \left(a^{1/n}\right)^m = \left(\sqrt[n]{a}\right)^m$

Numbers $27^{2/3} = \left(27^{1/3}\right)^2 = \left(\sqrt[3]{27}\right)^2$

Rewrite each in rational exponent form.

1. $\left(\sqrt[5]{4}\right)^3$

$4^{\frac{3}{5}}$

2. $\left(\sqrt[3]{-8}\right)^2$

$(-8)^{\frac{2}{3}}$

3. $\left(\sqrt[4]{15}\right)^7$

$15^{\frac{7}{4}}$

Rewrite each exponent in radical form.

1. $(-3)^{2/5}$

$\left(\sqrt[5]{-3}\right)^2$

2. $6^{3/2}$

$\left(\sqrt[2]{6}\right)^3$

$=\left(\sqrt{6}\right)^3$

3. $12^{3/4}$

$\left(\sqrt[4]{12}\right)^3$

In Exercises 1–6, find the indicated real *n*th root(s) of *a*.

1. $n = 2, a = 64$

$64^{\frac{1}{2}} = \boxed{\pm 8}$

2. $n = 3, a = 27$

$27^{\frac{1}{3}} = \boxed{3}$

3. $n = 4, a = 256$

$256^{\frac{1}{4}} = \boxed{\pm 4}$

4. $n = 5, a = 243$

$243^{\frac{1}{5}} = \boxed{3}$

5. $n = 8, a = 256$

$256^{\frac{1}{8}} = \boxed{\pm 2}$

6. $n = 4, a = 10{,}000$

$10{,}000^{\frac{1}{4}}$

$\boxed{= \pm 10}$

In Exercises 7–12, evaluate the expression.

7. $\sqrt[4]{625}$

$= 625^{1/4}$

$\boxed{= \pm 5}$

8. $\sqrt[3]{-512}$

$= (-512)^{1/3}$

$\boxed{= -8}$

9. $\sqrt[3]{-216}$

$= (-216)^{\frac{1}{3}}$

$\boxed{= -6}$

10. $\sqrt[5]{-243}$

$= (-243)^{1/5}$

$\boxed{= -3}$

11. $729^{1/6}$

$\boxed{= \pm 3}$

12. $(-81)^{1/2}$

no solution

Evaluate each expression.

1. $32^{2/5}$

   $= 4$

2. $343^{2/3}$

   $= 49$

3. $(-64)^{3/2}$

   no solution

4. $256^{7/8}$

   $\boxed{= \pm 128}$

5. $-729^{5/6}$

   no solution

6. $(-625)^{3/4}$

   no solution

Homework

6.1 Pg. 296 #17 - 22, 47, 49

6.2 Pg. 303 #4, 6 - 8, 14, 16, 24, 26