5.1 N^{th} roots & Rational Exponents

$2^3 = 8$

In general for an integer n greater than 1, if $b^n = a$, then b is an n^{th} root of a.

$$\sqrt[n]{a}$$

Because $a^{1/2}$ is a number whose square is a, you can write $\sqrt{a} = a^{1/2}$. Simarlarly $\sqrt[3]{a} = a^{1/3}$

$$\sqrt[4]{a} = a^{1/4}$$
$$\sqrt[5]{a} = a^{1/5}$$
$$\sqrt[n]{a} = a^{1/n}$$, for any integer greater than 1.

n is an even integer		n is an odd integer	
$a<0$	no real n^{th} roots	$a<0$	1 real n^{th} root $\sqrt[n]{a} = a^{1/n}$
$a=0$	One real n^{th} root $\sqrt[n]{0} = 0$	$a=0$	1 real n^{th} root $\sqrt[n]{0} = 0$
$a>0$	2 real n^{th} roots: $\pm\sqrt[n]{a} = \pm a^{1/n}$	$a>0$	1 real n^{th} root $\sqrt[n]{a} = a^{1/n}$

Ex. 1 Find the indicated real n^{th} root(s) of a.

1. $n = 3$, $a = -216$

n is odd, has 1 real n^{th} root

$\sqrt[3]{-216} = -216^{1/3}$

$(-6)^3 = -216$

-6

2. $n = 4$, $a = 81$

n is even $a>1$ 2 real n^{th} roots

$\sqrt[4]{81}$

9 9

3 3 3 3

$3^4 = 81$

$81^{1/4} = \pm 3$

5.1 ex. 1 cont.

practice

1.) $n = 4,\ a = 16$

$\sqrt[4]{16}$

even - 2 real solutions

$16^{1/4} = 2^4 = 16$

16: 4 4; 4: 2 2, 4: 2 2

± 2

even,

2) $n = 2,\ a = -49$

$\sqrt[2]{-49}$

$a < 0$

$\boxed{\text{no real } n^{th} \text{ roots}}$

3.) $n = 3,\ a = -125$

n-odd   1 root

$-125^{1/3} = \sqrt[3]{-125} = (-5)^3$

-5

4.) $n = 5 \quad a = 243$

1 real n^{th} term

$\sqrt[5]{243} = 243^{1/5} \rightarrow 3^5$

243: 9 27; 9: 3 3, 27: 9 3; 9: 3 3

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Ex. 2 Rational Exponents

$$a^{\frac{m}{n}} = (a^{1/n})^m = (\sqrt[n]{a})^m$$

$$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{(a^{1/n})^m} = \frac{1}{(\sqrt[n]{a})^m} \quad a \neq 0$$

Evaluate

a.) $16^{3/2}$

$(\sqrt[2]{16})^3 = (4)^3 = 64$

or

$(16^{1/2})^3 = (4)^3 = 64$

b.) $32^{-3/5} = \frac{1}{32^{3/5}} = \frac{1}{(\sqrt[5]{32})^3} = \frac{1}{2^3} = 1/8$

$32^{-3/5} = \frac{1}{32^{3/5}} = \frac{1}{(32^{1/5})^3} = \frac{1}{2^3} = 1/8$

Ex.3 Approx. Expressions w/rational Exponents.

a.) $9^{1/5} \approx 1.55$ b.) $12^{3/8} \approx 2.54$ c.) $(\sqrt[4]{7})^3$

$7^{3/4} \approx 4.30$

Monitor

Evaluate w/out a calculator.

5.) $4^{\frac{5}{2}} = (4^{1/2})^5$

$(\sqrt[2]{4})^5$

$(2)^5 = 32$

6.) $9^{-1/2}$

$\frac{1}{9^{1/2}} = \frac{1}{\sqrt[2]{9}}$

$= \frac{1}{3}$

7.) $81^{3/4}$

$(\sqrt[4]{81})^3$

$(81^{1/4})^3$

$(3)^3 = 27$

8.) $1^{7/8}$

$(1^{1/8})^7$

$= 1$

9.) $6^{2/5}$

≈ 2.05

10.) $64^{-2/3}$

$\frac{1}{64^{2/3}} \approx .06$

11.) $(\sqrt[4]{16})^5 = 16^{5/4}$

$= 32$

12.) $(\sqrt[3]{-30})^2$

$(-30)^{2/3}$

put in ( )

≈ 9.65

Ex.4 Solving Equations Using n^{th} roots.

Find the real solutions of...

(a.) $4x^5 = 128$

$\frac{4x^5}{4} = \frac{128}{4}$

$\sqrt[5]{x^5} = \sqrt[5]{32}$

n is odd
1 real soln

$x = 32^{1/5}$

$x = 2$

(b.) $(x-3)^4 = 21$

$\sqrt[4]{(x-3)^4} = \sqrt[4]{21}$

n - even
2 real
Soln.

$x - 3 = \pm\sqrt[4]{21}$

$+3 \quad +3$

$x = 3 \pm \sqrt[4]{21}$

$\boxed{3 + \sqrt[4]{21} \qquad 3 - \sqrt[4]{21}}$

$3 + 21^{1/4} \qquad 3 - 21^{1/4}$

$3 + 2.14 \qquad 3 - 2.14$

$x \approx 5.14$ and $x \approx 0.86$

HW: pg. 241 - 242 #6-44 even

6.) $n = 5$, $a = -1$

odd, one real solution

$(-1)^{1/5} =$

$\sqrt[5]{-1} = -1$

8.) $\sqrt[4]{256} = 256^{1/4}$

even, 2 roots

± 4

10.) $\sqrt[6]{-729}$

no real n^{th} root

12.) $8^{1/3} = \sqrt[3]{8}$ 1 solution

$8 = 4 \cdot 2$, $4 = 2 \cdot 2$

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